Decentralized LQ-Consistent Event-triggered Control over a Shared Contention-based Network

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***Abstract*—Consider a network of multiple independent stochastic linear systems where a scheduler collocated with the output sensors of each system arbitrates data transmissions to a corresponding remote controller through a shared contention-based communication network. While the systems are independent, their optimal controller design problem may generally become coupled, due to network contention, if the schedulers trigger transmissions based on state-dependent events. In this article we propose a class of probabilistic admissible schedulers for which the optimal controllers with respect to local standard LQG costs have the certainty equivalence property and can still be designed decentrally. Then, two scheduling policies within this class are introduced; a non-event-based and an event-based, where they both have an easily adjustable triggering probability at every time-step. The main contributions of this work are as follows: i) proving that for each system, the control loop with the event-based scheduler and its optimal controller outperforms the control loop with the non-event-based scheduler and its associated optimal controller; ii) showing that for each system, the local optimal state estimator for both scheduling policies follows a linear iteration; i) regulation of triggering probabilities of the schedulers by maximizing a network utility function.**

***Index Terms*—Shared contention-based communication network, Decentralized optimal LQG controller, LQ-Consistent event-based policy, Network utility maximization.**

I Introduction

Event-triggered control (ETC) pertains to strategies that manage transmissions in a control loop based on events rather than time and is intended for scenarios where communication resources are scarce or expensive. Extensive research has been carried out in the past decade on ETC, mostly focusing on a single control loop closed by a single communication channel [1]–[5]. However, communication management is especially interesting in settings where the communication network is shared by multiple control loops [6]–[9], as illustrated in Fig. 1. Contention-free protocols such as time-division multiple access (TDMA) [10] enable periodic transmission for all the control loops but lead to inefficient bandwidth usage when some of the control loops alternate between being active and inactive. In fact, under these conditions, systems operating through these protocols need a central coordinator for the resource reallocation between different users and therefore, they are not decentrally scalable. In turn, contention-based protocols such as slotted-ALOHA [10] are decentrally reconfigurable which facilitates the design scalability. However, contention in these protocols could hamper the control design analysis when the loops transmit based on events. In fact, if event-based data triggering is influenced by the control inputs, then due to network contention, the local optimal controllers are in general not globally optimal anymore and a centralized design strategy is needed to achieve optimality of control.

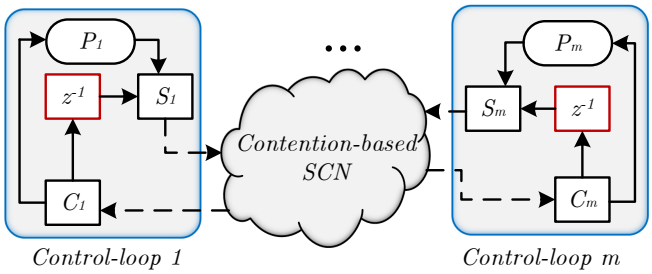

Figure 1: Networked control loops through a shared communication network (SCN); P describes the plant, S the scheduler, C the controller and z^{-1} a one time-step delay.

Motivated by this discussion, in this article we start by proposing a class of decentralized admissible schedulers for each control loop in a contention-based setting, within which the design of optimal control strategies are performed in a decoupled fashion. This class specifies that transmissions occur based on a constant function of primitive random variables, i.e. disturbance, noise and any other possible independent random variable. Then a subclass of these policies is introduced where its triggering probability is easily tunable. Naturally, a non-event-based purely stochastic transmission (PST) policy belongs to this class. Our main contribution is to propose an event-triggered controller, consisting of an event-based scheduling policy in this class and an optimal control policy with respect to an average quadratic cost, that outperforms the PST policy also with the associated optimal control input. Inspired by [11], we refer to this property as *LQ-consistency*. Moreover, we propose a method to regulate the schedulers

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based on their triggering probabilities in order to maximize a network utility function in the sense of providing a proportional fairness between the users and taking into account the control performance of each loop.

The event-based scheduler proposed in this work is inspired by the stochastic threshold event-triggered transmission (STETT) policy [11], [12] based on which data is triggered when the norm of the error between local and remote state estimators becomes larger than an exponentially distributed random threshold. We adapt this scheduling policy to the control loops of the contention-based communication network and show that the optimal control law, can still be found in a decentralized fashion based on local state estimators following the Kalman filter.

Related work: Comparing the performance of time-triggered and event-triggered scheduling policies over shared communication networks has received considerable attention in recent years [13]–[15]. In [13], it is shown, via a numerical example, that a threshold-based event-triggered scheduling policy with CSMA protocol results in a better performance, measured by the average state variance than a time-triggered policy with TDMA or FDMA protocols. Moreover, [14] considers the average state variance performance index and compares its values for different communication protocols and load conditions. Based on that, an event-based policy over ALOHA protocol performs better than a time-triggered policy over TDMA protocol for low communication loads, however, when the communication load is high, the performance of the event-triggered policy over ALOHA protocol becomes worse. Moreover, [15] claims that event-triggered policies for the state estimation may perform worse than time-triggered policies, in the sense of average state error covariance, if the effect of the communication network is explicitly considered. Some researchers also consider stability of the control loops when transmitting through a shared communication network [16]–[18]. In [16], the triggering probabilities of every scheduler transmitting through the CSMA protocol are determined in order to guarantee Lyapunov mean square stability (MSS). These triggering probabilities can be used to tune the threshold of event-based scheduling policies. Furthermore, an error-based dynamic priority allocation mechanism is employed for the network contention resolution of the event-triggered data of multiple dynamic users and then the stochastic stability of the overall network is proved in [17], [18]. Optimal co-design problem of an event-triggered scheduler and a controller is investigated in [19], [20] as in the current work. We extend the result of [19] for multiple control loops closed over a shared contention-based network. Establishing the optimality of the linear certainty equivalent controller when the event-based scheduler just depends on the primitive random variables, is the main contribution of [19]. Moreover, it is shown in [20] that the optimal event generator for data transmission of the minimum-variance output feedback control is the solution of an optimal stopping time problem. More related work can be found in [21]–[26].

Contribution: The main contributions of this article compared to its conference versions [27], [28] are as follows:

- Definition of a class of scheduling policies for the users of a contention-based communication network within which the decentralized design structure of the optimal controller is maintained.
- Establishment of the consistency of the proposed event-triggered control for time-varying triggering probabilities which is needed to maximize the network utility after every change in the number of the network users [29].
- Determination of the schedulers triggering probabilities by using the notion of network utility function.
- Investigation of the output feedback case, as extension to the results in [27] where it is assumed that the perfect state is available for every scheduler.

Organization: The remainder of this article is organized as follows: the problem of interest is introduced in Section II, the decentralized optimal control policy for the admissible schedulers is determined in Section III, the state estimator and the performance of the non-event-based PST is determined in Section IV, the novel event-based scheduling policy is introduced in Section V and the main results of the article are presented in Section VI. Moreover, we discuss the decentralized implementation of the proposed policies and the network utility maximization in Section VII. Finally, the effectiveness of the results is demonstrated through numerical simulations in Section VIII and Section IX presents some concluding remarks. The proofs of the lemmas and theorems can be found in the Appendix.

Notation: $f(x|y)$ denotes the conditional probability density function (pdf) of a random variable x given the set of information y and $\mathcal{N}(\bar{y}, Y)$ indicates a multi-variate Gaussian pdf with mean $\bar{y}$ and covariance Y. $\Pr(.)$ denotes the probability of an event; $\delta \sim \mathrm{B}(p)$ indicates that the random variable δ follows a Bernoulli distribution with probability p. By $\varrho(A)$ and $\mathrm{tr}(A)$ we denote the spectral radius and the trace of the square matrix A, respectively. Moreover, $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$ in which $\mathbb{N}$ is the set of natural numbers and $Z_t^s := \{k \in \mathbb{Z} | t \leq k \leq s\}$ where $\mathbb{Z}$ is the set of integers.

II Problem Setting

Consider a networked system comprised of multiple independent stochastic linear time-invariant (LTI) subsystems each modeled with the following discrete time dynamics

$$\begin{aligned} x_{k+1}^i &= A_i x_k^i + B_i u_k^i + w_k^i, \\ y_k^i &= C_i x_k^i + v_k^i \end{aligned} \tag{1}$$

in which $x_k^i \in \mathbb{R}^{n_i}$, $u_k^i \in \mathbb{R}^{m_i}$ and $y_k^i \in \mathbb{R}^{o_i}$ are, respectively, the state, the control input and the output vectors at time-step $k \in \mathbb{N}_0$ for $i \in Z_1^m$ where m is the total number of subsystems. Let $\{w_k^i | k \in \mathbb{N}_0\}$ and $\{v_k^i | k \in \mathbb{N}_0\}$ be sequences of i.i.d. Gaussian random variables with zero means and covariances $W_i = \mathbb{E}[w_k^i w_k^{i\intercal}]$ and $V_i = \mathbb{E}[v_k^i v_k^{i\intercal}]$ for every $k \in \mathbb{N}_0$. Moreover, the pairs (A_i, B_i) and (A_i, C_i) are assumed to be controllable and observable, respectively. The performance of each system is measured by the following local average quadratic cost

$$J^i = \limsup_{T \to \infty} \frac{1}{T} \mathbb{E}[\sum_{k=0}^{T-1} x_k^{i\intercal} Q_i x_k^i + u_k^{i\intercal} R_i u_k^i] \tag{2}$$

in which Q_i and R_i are positive semi-definite and positive definite matrices with appropriate dimension, respectively, and $(A_i, Q_i^{\frac{1}{2}})$ is assumed to be observable for every $i \in Z_1^m$. Therefore, each dynamic user is characterized by the tuple $(A_i, B_i, C_i, W_i, V_i, Q_i, R_i)$, which in general and typically is different from the other users, i.e. the users are heterogeneous.

As depicted in Fig. 1, we assume that the sensors of every subsystem are collocated with a scheduler that arbitrates data triggering in the control loop over the shared communication network. Moreover, since different subsystems are assumed to be physically independent, every scheduler just needs to transmit its locally acquired information to its corresponding remote controller. We also assume time-synchronization of the sampling process of different subsystems where they all have equal sampling time. Besides the performance index (2) for each individual control loop, one may also define a social index such as $\sum_{i=1}^m J^i$ or a network utility function as suggested in [29]. This will be discussed in Sections IV and VII.

In the remainder of this section, we introduce the operation mechanism of the shared communication network, the information structure of the scheduler and the controller of every control-loop and the main characteristics of the scheduling policies which admit a decentralized control design structure. This section will be concluded with the problem statement of our interest.

II-A Assumptions on the shared communication network

The assumptions on the shared communication network are very close to the ones considered in the context of the contention-based protocols [10] and can be summarized as follows:

- Time is partitioned into fixed-size slots; during each time-slot only one user can transmit successfully.
- All users are restricted to start a new transmission at the beginning of the time-slot.
- Collision will occur if more than one user attempt to transmit which results in loss of the collided data.
- Every user should receive a data receipt acknowledgment at the same time-slot if data transmission is successful, otherwise, it is assumed that a collision has occurred.

Besides these assumptions, we make two additional assumptions suited to control applications:

- There is no retransmission mechanism for the collided data.
- The transmission time is assumed to be negligible with respect to the duration of the time-slot, i.e., the controller is assumed to access data at the beginning of the time-slot when a successful transmission occurs.

The first of these two assumptions is different from the standard retransmission mechanism of contention-based protocols and is motivated by the fact that retransmissions in control applications would result in long delays and buffering new data instead of transmitting it immediately (see [14, Theorem 13]). Therefore, we consider that after a collision/data drop, the newest sensor information can be obtained and a new transmission with fresh data can be attempted. As we shall see shortly, the schedulers will be restricted such that this new transmission attempt occurs with a given probability at every time-slot, as this is the case in the retransmission mechanism of slotted-ALOHA, except that new data is sent in our setting. Moreover, implicit in the second assumption is the fact that we consider the time-step of the discrete-time system k to coincide with the time-slot index, i.e., x_k^i pertains to the state of the system at the beginning of the time-slot.

II-B Information structure

In this section, we introduce the information set available for the scheduler and the controller at every time-step. Let

$$\delta_k^i = \begin{cases} 1, & \text{if the scheduler } i \text{ attempts data transmission at } k \\ 0, & \text{otherwise} \end{cases}$$

and

$$\rho_k^i = \begin{cases} 1, & \text{if the network is available for the user } i \text{ at } k \\ 0, & \text{otherwise} \end{cases}$$

for every user $i \in Z_1^m$ at every time-step $k \in \mathbb{N}_0$. Based on the properties of the shared contention-based communication network introduced in Section II-A, we can write

$$\rho_k^i = \prod_{j=1, j\neq i}^{m} (1 - \delta_k^j).$$

Moreover, let $\sigma_k^i = \rho_k^i \delta_k^i$ be a variable indicating a successful transmission at every time-step, in which case $\sigma_k^i = 1$ and $\sigma_k^i = 0$, otherwise. Note that based on the structure of the shared communication network in Section II-A, every scheduler receives an error-free acknowledgment signal from the controller whenever an attempted transmission is successful, and therefore it knows all the previous values of ρ_k^i. Accordingly, in order to decide on δ_k^i the scheduler has the following information set at a time-step k

$$\mathcal{I}_k^i = \{\delta_\ell^i, y_\ell^i | \ell \in Z_0^{k-1}\} \cup \{\rho_\ell^i | \delta_\ell^i = 1 \wedge \ell \in Z_0^{k-1}\} \cup \{y_k^i\}. \quad (3)$$

Let $\hat{x}_{k|k}^i = \mathbb{E}[x_k^i | \mathcal{I}_k^i]$ and $\hat{x}_{k+1|k}^i = \mathbb{E}[x_{k+1}^i | \mathcal{I}_k^i]$ be the state estimations of the current and the next time-step computed by the local state estimator collocated with the scheduler. We know that the optimal least-square state estimator at the scheduler is linear and can be computed recursively by the Kalman filter as follows

$$\begin{aligned} \hat{x}_{k+1|k}^i &= A_i \hat{x}_{k|k}^i + B_i u_k^i, \\ \hat{x}_{k|k}^i &= \hat{x}_{k|k-1}^i + L_i (y_k^i - C_i \hat{x}_{k|k-1}^i) \end{aligned} \quad (4)$$

where

$$\begin{aligned} L_i &= \Theta_i C_i^\intercal (C_i \Theta_i C_i^\intercal + V_i)^{-1}, \\ \Theta_i &= A_i \Theta_i A_i^\intercal + W_i - A_i L_i (C_i \Theta_i C_i^\intercal + V_i) L_i^\intercal A_i^\intercal. \end{aligned} \quad (5)$$

For simplicity we assume $\mathbb{E}[(x_0^i - \hat{x}_{0|0}^i)(x_0^i - \hat{x}_{0|0}^i)^\intercal] = \Theta_i$ which implies $\mathbb{E}[(x_k^i - \hat{x}_{k|k}^i)(x_k^i - \hat{x}_{k|k}^i)^\intercal] = \Theta_i,\ \forall k \in \mathbb{N}_0$ since Θ_i is the fixed point of the Kalman filter's time-varying Ricatti equation.

When the triggering condition is satisfied, i.e. $\delta_k^i = 1$, the scheduler transmits state information over the communication network to be received by its corresponding controller. A

natural choice would be transmitting all the newly acquired output vectors over the time period starting from the last successful transmission up to the current time-step. However, in this case, the information set available to the remote state estimator is the same as the one available to the local state estimator. This means that the optimal remote state estimation in case of a successful data transmission is equal to the one obtained by the Kalman filter in the scheduler [30]. Therefore, the scheduler can transmit $\hat{x}^i_{k|k}$ which is an equivalent and a more compact form to all the output values not transmitted so far. Accordingly, the information set available for the controller at every time-step k is as follows

$$\mathcal{J}^i_k = \{\hat{x}^i_{\ell|\ell} | \sigma^i_\ell = 1 \wedge \ell \in Z^k_0\} \cup \{\sigma^i_\ell | \ell \in Z^k_0\}. \tag{6}$$

II-C Required characteristics for the scheduling policies

We define next a class of admissible schedulers for which it will be shown (in Section III) that not only the optimal controller has the certainty equivalent property, but also it can be computed decentrally for the users of the shared contention-based communication network. Let us denote all independent random variables x^i_0, w^i_k, v^i_k of any control loop at all time-steps as the primitive random variables.

Definition 1: (*Admissible schedulers*) Any scheduler which depends only on the primitive random variables of its corresponding control loop, i.e.

$$\delta^i_k = g(x^i_0, w^i_{0:k-1}, v^i_{0:k}, a^i_{0:k})$$

where the triggering law g is fixed and $a^i_{0:k}$ represents all the other independent random variables of the same control loop, is called an admissible scheduler. □

Let us refer to a subclass of the admissible schedulers with an easily tunable triggering probability as *tunable admissible schedulers*. Moreover, assume that the transmission mechanism of each user of the contention-based communication network is based on a tunable admissible scheduler with a given triggering probabilities $p^i_k,\ \forall i \in Z^m_1$ at every time-step. Then from a single control loop perspective, the contention-based communication network can be abstracted as if at every time-step there is a probability

$$q^i_k = \prod_{j=1, j\neq i}^{m} (1 - p^j_k) \tag{7}$$

that all the other users are not trying to transmit and the network is therefore available. Hence, at every time-step, the control loop of interest has a successful transmission probability of

$$\eta^i_k = q^i_k p^i_k \tag{8}$$

which directly affects stability and the performance characteristics of that control loop, and also the overall networked system.

II-D Problem statement

In this section, we first introduce some concepts and then state the problem to be tackled in this article. When every tunable admissible scheduler of the network uses the information set defined in (3) for deciding on data transmission, then we call the scheduling policy an event-triggered or event-based. Otherwise, i.e., if the tunable admissible scheduler operates in a non-event-based fashion with a given triggering probability at every time-step, then transmissions happen purely stochastically as defined next:

Definition 2: (*Purely stochastic policy*) For any control loop of the contention-based communication network, a non-event-based tunable admissible scheduler operates by triggering data purely stochastically, i.e.

$$\delta^{i,ps}_k \sim \mathrm{B}(p^i_k), \tag{9}$$

where p^i_k is a given triggering probability at every time-step $k \in \mathbb{N}_0$. This transmission mechanism is referred to as purely stochastic transmission (PST) policy. □

The PST is a suitable non-event-based tunable admissible scheduler for transmitting through the contention-based networks. We will show in the sequel (see Theorem 1 and Lemma 1) that given the PST scheduling policy for any control loop of the network one can compute its optimal control law analytically. Now inspired by [31], let us define the following LQ-consistency property for any possible event-triggered control policies:

Definition 3: (*LQ-Consistency*) An event-triggered control policy of any control loop is called LQ-consistent if it results in a better performance, measured by the average quadratic cost (2), than that of the control loop with the PST policy and its associated optimal controller. □

Then the problem we are interested in can be stated as follows: Find an LQ-consistent decentralized event-triggered control policy suitable for the users of the contention-based communication network.

III Decentralized optimal control design

In this section, we establish that when the schedulers of the contention-based network users are admissible according to Definition 1, then the optimal control design problem has the certainty equivalence property and is equivalent to the design of several locally optimal controllers. We need the following assumption in future analysis.

Assumption 1: All the schedulers of the control loops of the shared contention-based communication network are admissible according to Definitions 1. □

Theorem 1: Let Assumption 1 hold and the control signal u^i_k is generated according to the following state feedback law

$$u^i_k = K_i \mathbb{E}[x^i_k | \mathcal{J}^i_k] \tag{10}$$

for every $i \in Z^m_1$ where

$$\begin{aligned} K_i &= -(B_i^\intercal P_i B_i + R_i)^{-1} B_i^\intercal P_i A_i, \\ P_i &= A_i^\intercal P_i A_i + Q_i - K_i^\intercal (B_i^\intercal P_i B_i + R_i) K_i. \end{aligned} \tag{11}$$

Then the control policy (10) is optimal if the loop is MSS, i.e., $\sup\{\mathbb{E}[x^i_k x^{i\intercal}_k]\} \leq c$ for a $c \in [0, \infty)$ and $k \in \mathbb{N}_0$. □

Proof of Theorem 1: Let us consider the whole networked control system centrally and denote $\mathcal{J}_k = \cup^m_{i=1} \mathcal{J}^i_k$ as the total information available for the hypothetical central controller at

every time-step. First of all, we have to establish the certainty equivalence property, i.e., that the control law takes the form

$$u_k^i = K_i \mathbb{E}[x_k^i | \mathcal{J}_k], \tag{12}$$

where due to physical independency between different dynamic users, the optimal control gain for every system can be determined independently, as given in (11). As explained in [32], we can conclude certainty equivalence property if we show the following two features:

- Independency of the scheduling law from the control inputs which prevents the generation of the control inputs' dual effect in the control loops.
- Independency of the remote state estimation error from the control inputs.

Since every δ_k^i follows a constant function of the primitive random variables, any $\sigma_k^i = \delta_k^i \prod_{j=1,j\neq i}^m (1-\delta_k^j)$ is independent from all control inputs which indicates that the first required property holds. Moreover, following the same steps as in [30], we can prove that the state estimation errors, i.e. $e_{k|k}^i = x_k^i - \mathbb{E}[x_k^i|\mathcal{J}_k]$, for both forced and unforced dynamics with the same realization of the primitive random variables are equal, therefore, the state estimation error in the controller is independent from the control inputs. Accordingly, the optimal control law for every system follows (12).

Now we have to show that $\mathbb{E}[x_k^i|\mathcal{J}_k] = \mathbb{E}[x_k^i|\mathcal{J}_k^i]$ holds for all dynamic users at every time-step. Based on the Bayes law of conditional probability, we have the following equality at all time-steps and for all dynamic users

$$f(x_k^i|\mathcal{J}_k^i, \cup_{j=1,j\neq i}^m \mathcal{J}_k^j) = \frac{\Pr(\cup_{j=1,j\neq i}^m \mathcal{J}_k^j | \mathcal{J}_k^i, x_k^i)}{\Pr(\cup_{j=1,j\neq i}^m \mathcal{J}_k^j | \mathcal{J}_k^i)} f(x_k^i|\mathcal{J}_k^i)$$

where it is clear that the fraction term in the above equation is equal to one and $f(x_k^i|\mathcal{J}_k^i, \cup_{j=1,j\neq i}^m \mathcal{J}_k^j) = f(x_k^i|\mathcal{J}_k^i)$ which concludes the statement. Therefore, the optimal local controllers are equivalent to the optimal central controller. ■

Based on Theorem 1, the optimal control and the remote state estimation problems admit a decentralized structure when all the schedulers are admissible according to Definition 1. Accordingly, we can analyze the controller of every closed-loop system independently from the others as depicted in Fig. 2. Furthermore, in Sections IV and V, we establish that for the PST and an event-based transmission policies the state estimation required in (10) is computed by a local Kalman filter.

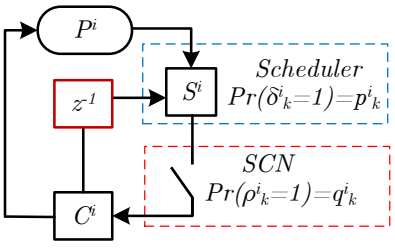

Figure 2: A decoupled control loop of the contention-based shared communication network (SCN).

IV Purely Stochastic Transmission Policy

In this section, we analyze every control loop of the shared contention-based communication network where its scheduler is operating based on the PST policy introduced in Definition 2. First, it will be shown that given Assumption 1, the remote state estimation of this control loop follows a linear iteration (Kalman filter). Second, the closed-form expression of the local average quadratic performance is determined when all the schedulers have constant triggering probabilities at every time-step.

Lemma 1: Consider that the Assumption 1 holds, then for every control loop with the scheduler operating based on the PST policy (9), we have

$$\begin{aligned} \bar{x}_{k+1|k}^i &= A_i \bar{x}_{k|k}^i + B_i u_k^i, \\ \bar{x}_{k|k}^i &= \begin{cases} \hat{x}_{k|k}^i, & \text{if } \sigma_k^i = 1, \\ \bar{x}_{k|k-1}^i, & \text{otherwise}, \end{cases} \end{aligned} \tag{13}$$

at every time-step where $\bar{x}_{k|k}^i = \mathbb{E}[x_k^i|\mathcal{J}_k^i]$ is the remote state estimation needed for the calculation of the optimal control policy in (10). □

The proof of Lemma 1 is available in the Appendix.

The MSS and the performance of the control loops of the shared contention-based communication network depend highly on the data transmission probabilities. The following lemma provides the MSS condition and the optimal performance of every subsystem when the scheduler is operating based on the PST policy.

Lemma 2: Consider that the Assumption 1 holds and all the schedulers are transmitting with constant probabilities at every time-step, i.e. $p_k^i = p^i$, $\forall i \in Z_1^m$ and at all $k \in \mathbb{N}_0$. Then every control loop with the PST policy is MSS if

$$\varrho(\sqrt{1-q^i p^i} A_i) < 1 \tag{14}$$

where q^i is determined by (7), and the optimal average quadratic performance of subsystem i is

$$\begin{aligned} J_{ps}^i = \operatorname{tr}(P_i W_i) &+ \sum_{j=0}^{\infty} (1-q^i p^i)^{j+1} \operatorname{tr}(A_i^j W_i A_i^{j\intercal} Y_i) \\ &+ \sum_{j=0}^{\infty} q^i p^i (1-q^i p^i)^j \operatorname{tr}(A_i^j \Theta_i A_i^{j\intercal} Y_i) \end{aligned} \tag{15}$$

where $Y_i = K_i^\intercal (B_i^\intercal P_i B_i + R_i) K_i$, and Θ_i is given in (5). □

The proof of Lemma 2 is available in the Appendix.

A key question is how to find the optimal values of the triggering probabilities in order to minimize a social performance index. The natural answer might be to consider the following optimization problem

$$(p^1, \ldots, p^m)^* = \arg \min_{p^1,\ldots,p^m} \sum_{i=1}^m J_{ps}^i. \tag{16}$$

However, (16) is in general a non-convex and non-separable and therefore, not easily solvable. In Section VII, we determine the constant triggering probabilities of the schedulers based on a network utility maximization criterion which is more tractable.

V Proposed scheduling policy using stochastic thresholds

The novel tunable admissible scheduler proposed in this work combines the features of the STETT [11] and the PST policies. In this section, we first introduce the STETT policy and discuss its advantages. Then, we propose the combined scheduling policy which is in the class of tunable admissible schedulers.

V-A Stochastic Threshold Event-triggered Transmission

Inspired by [11], the data triggering mechanism of the STETT policy for every linear system with Gaussian disturbance and noise is defined as follows

$$\delta_k^{i,st} = \begin{cases} 1, & \text{if } \frac{1}{2} e_{k|k-1}^{i\intercal} \Psi_{k|k-1}^{i,-1} e_{k|k-1}^i > r_k^i, \\ 0, & \text{otherwise}, \end{cases} \tag{17}$$

in which $r_k^i \sim \exp(\lambda_k^i)$ for $\lambda_k^i \in \mathbb{R}_{\geq 0}$ is an exponentially distributed random threshold, $e_{k|k-1}^i = \hat{x}_{k|k}^i - \bar{x}_{k|k-1}^i$ and $e_{k|k}^i = \hat{x}_{k|k}^i - \bar{x}_{k|k}^i$ are the predicted and the updated state estimation errors between the local and the remote state estimations with the covariances as $\Psi_{k|k-1}^i = \mathbb{E}[e_{k|k-1}^i e_{k|k-1}^{i\intercal} | \mathcal{I}_k^i]$ and $\Psi_{k|k}^i = \mathbb{E}[e_{k|k}^i e_{k|k}^{i\intercal} | \mathcal{I}_k^i]$, respectively. At every time-step, the value of $e_{k|k-1}^i$ can be determined by subtracting the updated state estimation by the Kalman filter (4) and the predicted state estimation as given in (13).

Unlike deterministic threshold event-triggered transmission policies, STETT policy keeps $e_{k|k-1}^i$ Gaussian distributed at all time-steps when there is no data collision or drop-out [12]. However, as we shall see shortly, although after a successful transmission $e_{k|k-1}^i$ is Gaussian distributed until the first new attempt to transmit, in case of data collision in the current transmission attempt, the distribution of the state error will become the sum of two Gaussians at the following time-step.

To clarify this statement, note that the state estimation error used in the scheduling law (17) has the following dynamics

$$e_{k+1|k}^i = (1 - \sigma_k^i) A_i e_{k|k}^i + L_i (C_i \hat{e}_{k+1|k}^i + v_{k+1}^i) \tag{18}$$

where $\hat{e}_{k+1|k}^i = x_k^i - \hat{x}_{k|k-1}^i$ is the state estimation error of the Kalman filter which has a Gaussian distribution with zero mean and Θ_i as its covariance matrix for all time-steps. It can be shown that the state estimation error dynamics in (18) depends only on the following information set at every time-step k,

$$\mathcal{R}_k^i = \{v_t^i, w_t^i, r_t^i, \rho_t^i | t \in Z_0^{k-1}\} \cup \{v_k^i\} \cup \{x_0^i\}$$

which indicates that the SETT policy (17) is a function of the primitive random variables and therefore, can be categorized as an admissible scheduler according to the Definition 1. Shortly we will see that the triggering probability of the STETT policy can be easily tuned when there is no data collision or loss in the communication network.

Let $\sigma_{k-1}^i = 1$, then $e_{k|k-1}^i = L_i(C_i \hat{e}_{k|k-1}^i + v_k^i)$, which is clearly Gaussian. Therefore, assume that the distribution of the predicted state error is Gaussian at time-step k as it is the case if $\sigma_{k-1}^i = 1$, but with an arbitrary covariance $\Psi_{k|k-1}^i$, i.e.

$$f(e_{k|k-1}^i | \mathcal{I}_k^i) = \mathcal{N}(0, \Psi_{k|k-1}^i). \tag{19}$$

Then the next lemma shows that the pdf of the predicted state estimation error at the following time-step, i.e., $k+1$, is Gaussian in case of no data triggering ($\delta_k^{i,st} = 0$) and in case of a data triggering and collision ($\delta_k^{i,st} = 1 \wedge \rho_k^i = 0$) the state estimation error becomes sum of two Gaussians.

Lemma 3: Assume that the distribution of the predicted state error follows (19) at time-step k, then

$$p_k^i = \Pr(\delta_k^{i,st} = 1 | \mathcal{I}_k^i) = 1 - (1 + \lambda_k^i)^{-\frac{n_i}{2}} \tag{20}$$

is the probability of the data transmission by the STETT scheduler (17) at time-step k in which n_i is the dimension of the state vector. Moreover,

$$f(e_{k+1|k}^i | \delta_k^{i,st} = 0, \mathcal{I}_k^i) = \mathcal{N}(0, \hat{\Psi}_{k+1|k}^i) \tag{21}$$

and

$$\begin{aligned} &f(e_{k+1|k}^i | \delta_k^{i,st} = 1, \rho_k^i = 0, \mathcal{I}_k^i) \\ &\quad = \frac{1}{p_k^i} \mathcal{N}(0, \Psi_{k+1|k}^i) - \frac{1 - p_k^i}{p_k^i} \mathcal{N}(0, \hat{\Psi}_{k+1|k}^i) \end{aligned} \tag{22}$$

are the pdfs of the predicted state estimation error at time-step $k+1$ in case of no data triggering ($\delta_k^{i,st} = 0$) and data collision ($\delta_k^{i,st} = 1 \wedge \rho_k^i = 0$), respectively, where

$$\begin{aligned} \Psi_{k+1|k}^i &= A_i^\intercal \Psi_{k|k-1}^i A_i + \Phi_i, \\ \hat{\Psi}_{k+1|k}^i &= \frac{1}{1 + \lambda_k^i} A_i^\intercal \Psi_{k|k-1}^i A_i + \Phi_i, \end{aligned} \tag{23}$$

in which $\Phi_i = A_i^\intercal \Theta_i A_i - \Theta_i + W_i$. □

The proof of Lemma 3 is available in the Appendix.

According to Lemma 3, the distribution of the state estimation error ($e_{k|k-1}^i$) remains Gaussian over the time period between the last successful transmission and the first subsequent data collision. Moreover, within this time interval, based on (20) the triggering probability depends only on the random threshold parameter λ_k^i and thetefore, we can easily regulate it as follows

$$\lambda_k^i = 1 - (1 - p_k^i)^{-\frac{2}{n_i}}. \tag{24}$$

However, when a collision happens, the distribution of the state error becomes the sum of two Gaussians. More specifically, it can be shown that in between every two successive successful transmissions, every collision doubles the number of Gaussian terms of the state error pdf [28]. When the number of Gaussian terms is more than one, the triggering probability depends not only on the threshold parameter but also on the covariances of the multiple Gaussian terms. Therefore, it is not trivial to regulate the triggering probability desirably after the first collision instance. This motivates the proposed scheduling policy discussed in the next section.

V-B Combined Event-triggered Transmission Policy

In this section, we propose a combined event-triggered transmission (CETT) policy $\pi^i = (\mu_0^i, \mu_1^i, \mu_2^i, \dots)$, where $\mu_k^i : \mathcal{I}_k^i \to \{0, 1\}$ and $\delta_k^{i,\mu} = \mu_k^i(\mathcal{I}_k^i)$, which inherits the advantages of the STETT policy and is in the class of tunable admissible schedulers. Based on this policy, after every successful transmission, the scheduler triggers based on the STETT policy with any desired probability p_k^i up to the

time-step at which the first collision happens. After that, the scheduler keeps triggering based on the PST policy with the desired probability p_k^i until the next successful transmission time. This process is repeated between every two successive successful transmissions.

Definition 4: (*CETT*) Let $\bar{\ell}_k^i := \max\{\ell < k | \sigma_\ell^i = 1\}$ be the time of the last successful transmission before the current time-step k. Then, we can specify the CETT policy $\delta_k^{i,\mu}$ as follows

$$\delta_k^{i,\mu} = \begin{cases} \delta_k^{i,st}, & \text{if } k = \bar{\ell}_k^i + 1 \\ & \text{or if } (\delta_{\bar{\ell}_k^i+1}^{i,st}, \ldots, \delta_{k-1}^{i,st}) = (0, \ldots, 0) \\ \delta_k^{i,ps}, & \text{otherwise,} \end{cases} \quad (25)$$

where $\delta_k^{i,st}$ follows (17) with λ_k^i determined by (24) for a given p_k^i as the triggering probability and $\delta_k^{i,ps} \sim \mathrm{B}(p_k^i)$ for all time-steps. □

In Fig. 3, different admissible schedulers introduced in this work are categorized based on the tunability of their triggering probability. We know that the PST policy is always easily tunnable. However, the STETT policy is only easily tunnable after every successful data transmission time-step up to the next data triggering time-step. Therefore, we can say it is partially in the class of the tunable admissible schedulers. The CETT policy in (25) is proposed to follow the SETT policy as long as it is easily tunable, otherwise, it switches to follow the PST policy. The hatched region in the figure indicates the CETT policy.

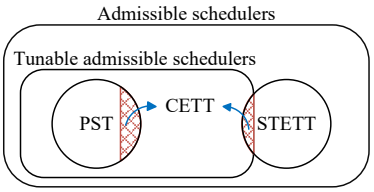

Figure 3: Relations between different admissible schedulers for the contention-based network.

VI Main results

In this section, we state the main results of the article. In the beginning, we introduce the optimal control policy associated with the CETT policy (25) in Theorem 2. Then we define the combined event-triggered control (CETC) policy in Definition 5. Finally, we establish the consistency property in Theorem 3 by showing that the CETC policy outperforms the PST policy when performance is measured by the average of local LQG cost functions.

Theorem 2: Let Assumption 1 hold and for a control loop the scheduler follow the CETT policy with a given set of triggering probabilities $\mathbf{P}^i = \{p_k^i | k \in \mathbb{N}_0\}$ and the controller follow (10) with the LQG gain in (11) and the linear state estimation in (13). This control policy is optimal if the loop is MSS, i.e., $\sup\{\mathbb{E}[x_k^i x_k^{i\intercal}]\} \leq c$ for a $c \in [0, \infty)$ and $k \in \mathbb{N}_0$. □

The proof of Theorem 2 is available in the Appendix. As it can be seen in the proof of Theorem 2, the pdf of the remote state estimation associated with the CETT policy (25) is Gaussian at every successful transmission time-step. However, it follows a sum of Gaussians where the number of Gaussian terms is two at the first time-step after a successful transmission and is increased by two every time-step elapses with no successful data transmission, i.e.,

$$\kappa_k^i = \begin{cases} 1, & \text{if } \sigma_k^i = 1 \\ 2, & \text{if } \sigma_k^i \neq 1 \wedge \bar{l}_k^i = k-1 \\ \kappa_{k-1}^i + 2, & \text{otherwise.} \end{cases}$$

where κ_k^i is the number of Gaussian terms of the remote state estimation error pdf at time-step k. It is worth noting that, these Gaussian terms all have the same mean values determined according to (13).

Definition 5: (*CETC*) The combination of the CETT policy (25) and its corresponding optimal control law (10), (11) and (13) is denoted by the CETC policy. □

We state next the main result of the article.

Theorem 3: Let Assumption 1 hold and a control loop be MSS when its scheduler follows the PST policy for a given set of triggering probabilities $\mathbf{P}^i = \{p_k^i | k \in \mathbb{N}_0\}$ and its controller follows the optimal policy (10) determined by (11) and (13). Then, the average quadratic performance (2) of this control loop when its scheduler-controller is operating based on the CETC policy strictly outperforms the optimal control performance when the scheduler is operating based on the PST policy, i.e.,

$$J_\pi^i < J_{ps}^i$$

when the set of triggering probabilities for both scheduling policies follows $\mathbf{P}^i$. □

The proof of Theorem 3 is available in the Appendix.

Remark 1: Based on Theorem 3, the MSS of each control loop with the scheduler following the PST policy for a given set of triggering probabilities $\mathbf{P}^i = \{p_k^i | k \in \mathbb{N}_0\}$ and its associated optimal controller can also guarantee the MSS when the control loop is operating based on the CETC policy.

VII Decentralized Implementation and Network utility maximization

From the discussions so far in this article, it can be concluded that all the schedulers and the control policies only require local information and therefore, can be implemented in a decentralized fashion. In this section, we first discuss how to regulate the schedulers to optimize a social criterion by determining the triggering probabilities, and then we show how to implement this optimization decentrally.

Determining the triggering probabilities of the schedulers based on the total average quadratic performance as in (16), results in a non-convex and a non-separable optimization problem, which is difficult to solve, especially in a distributed way. Therefore, we take a notion of network utility from [29] which considers a weighted proportional fairness between the network users. Based on that, for every user, we assume a constant triggering probability at every time-step (therefore, we drop its time index k for simplicity) and assign to it a utility allocation function as follows

$$U^i(\eta^i) = c^i \log(\eta^i) \quad (26)$$

where η^i defined in (8) is the constant successful transmission probability and the constant $c^i \in \mathbb{R}_{>0}$ determines the transmission priority of every user which can be selected based on the average quadratic performance of every control loop (15). Then the optimal successful transmission probabilities are determined as follows

$$(\eta^{1*}, \ldots, \eta^{m*}) = \arg \max_{\eta^1,\ldots,\eta^m} \sum_{j=1}^{m} U^j(\eta^j). \tag{27}$$

The following lemma, proposes the solution of this problem.

Lemma 4: The solution of (27) results in

$$p^{i*} = \frac{c^i}{\sum_{j=1}^{m} c^j} \tag{28}$$

for every $i \in Z_1^m$ as the optimal triggering probability of the schedulers of the contention-based communication network introduced in Section II-A. □

The proof of Lemma 4 is available in the Appendix. Moreover, we propose the following methods for the selection of $c^i,\ i \in Z_1^m$:

(i) The coefficient c^i for every $i \in Z_1^m$ can be selected based on the parameters which affect the average quadratic performance of the corresponding system. According to Lemma 2, a natural choice is

$$c^i = \alpha_i \text{tr}(A_i W_i A_i^\intercal Y_i) + (1-\alpha_i)\text{tr}(A_i \Theta_i A_i^\intercal Y_i)$$

where $\alpha_i \in [0,1]$ can be selected arbitrarily.

(ii) We can also select all transmission priorities to be equal which results in $p^{i*} = 1/m$ for all $i \in Z_1^m$. This triggering probability is equal to the optimal triggering probability for maximizing the throughput of the slotted-ALOHA communication channel [10].

Both strategies for tuning the triggering probabilities can be implemented in a decentralized fashion, as long as every node in the network has access to c^i of all nodes. This parameter can be either broadcasted at every time-step the node is active or just broadcasted once when a node joins the network (in a successful transmission), and when a node leaves the network a terminal message should also be sent such that each active node is informed which node has left the network.

VIII Numerical Simulations

In this section, we illustrate via a numerical example that the proposed CETC policy performs indeed better in comparison to the PST policy and we assert the performance gains. Consider a scalar LTI subsystem with $A = 0.9$, $B = 1$, $C = 1.5$, $D = 0$, $W = 1$ and $V = 1.5$. Due to the decentralized structure of the proposed policies, we can consider just a single control loop of this networked control system and drop the index i of the control loop parameters. Moreover, let $Q{=}1$ and $R{=}0.1$ be the parameters of the average quadratic performance. Then, the state feedback controller and the Kalman filter gains are $K = -0.8233$ and $L = 0.4476$, respectively. We consider a constant triggering probability for the scheduler of this control loop at all time-steps. In Fig. 4, we compare the average quadratic performance of both policies for two different constant probabilities that the network is

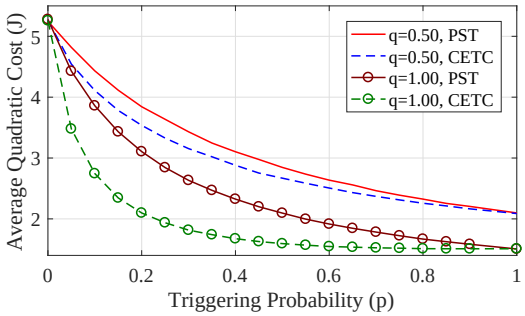

Figure 4: Average quadratic performance comparison between the PST and the CETC policies

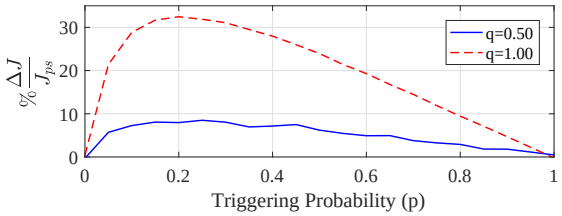

Figure 5: Average quadratic performance gain percentage of the CETC policy in comparison with the PST policy

free $q \in \{0.50, 1.00\}$ for this control loop at all time-steps. These plots illustrate what we have observed in Monte-Carlo simulations. For each pair of (p, q), we consider $n_{MC} = 10$ as the number of Monte-Carlo runs where for each of them, $T = 100000$ is the total number of simulation time-steps. The initial state for all simulations is assumed to be zero, i.e. $x_0 = 0$. Fig. 5 shows the percentage of the performance gains of the CETC policy with respect to the purely stochastic policy, i.e. $\%(J_{ps} - J_\pi)/J_{ps}$. As can be seen, when the availability probability of the network (q) for a specific control loop is higher, the performance gain obtained by the CETC policy is also higher.

IX Conclusion

This work considers multiple independent linear systems communicating through a shared contention-based communication network with their local remote controllers. We introduce a class of admissible schedulers which provides a decoupled optimal control design structure for the users of the shared contention-based communication network and is proved to have the certainty equivalence property. Then, two scheduling policies in this class of admissible schedulers are introduced, a non-event-based and an event-based policies where their triggering probabilities are easily tunable at every time-step. This feature can be used for maximizing the utility of the network in the sense of providing a proportional fairness between the users of the network. It is proved that for both of these two scheduling policies, the local optimal control law is determined based on a Kalman filter state estimator. The main contribution of this article is the LQ-consistency of the proposed event-based control strategy, i.e. for any system, the loop with the event-based scheduler and its optimal control

law outperforms the non-event-based scheduling policy with its associated optimal control law, as the triggering probability of both scheduling policies are the same at every time-step.

APPENDIX

A Lemma 1

We assume that the local estimator is aware of the control law and since $\mathcal{J}^i_{k-1} \subseteq \mathcal{I}^i_k$, then the local estimator is aware of all the previous control input values, i.e. $\{u^i_t | t < k\}$ at every time-step k. As discussed before, the information set of the local and the remote estimator is equivalent at triggering time-steps, therefore, $\bar{x}^i_{k|k} = \hat{x}^i_{k|k}$ when $\sigma^i_k = 1$. However, when $\sigma^i_k = 0$, then the remote state estimation pdf is as follows

$$f(x^i_k|\mathcal{J}^i_{k-1}, \sigma^i_k = 0) = \frac{\Pr(\sigma^i_k = 0|\mathcal{J}^i_{k-1}, x^i_k)}{\Pr(\sigma^i_k = 0|\mathcal{J}^i_{k-1})} f(x^i_k|\mathcal{J}^i_{k-1}).$$

Since the unsuccessful transmission probability is independent from x^i_k, the fraction term in the right hand side of the above equation is equal to one and $f(x^i_k|\mathcal{J}^i_{k-1}, \sigma^i_k = 0) = f(x^i_k|\mathcal{J}^i_{k-1})$ which indicates that in case of unsuccessful transmission, it is just needed to perform the prediction stage of the Kalman filter, i.e. $\bar{x}^i_{k|k} = \bar{x}^i_{k|k-1}$ where $\bar{x}^i_{k+1|k} = A_i \bar{x}^i_{k|k} + B_i u^i_k$. Therefore, the conditional state expectation in the remote estimator is proved to be given by (13).

B Lemma 2

We can represent the Kalman filter as

$$\hat{x}^i_{k+1|k+1} = A_i \hat{x}^i_{k|k} + B_i u^i_k + L_i(y^i_{k+1} - C_i \hat{x}^i_{k+1|k}) \quad \text{(A.1)}$$

and the remote state estimator as

$$\bar{x}^i_{k+1|k} = \sigma^i_k A_i \hat{x}^i_{k|k} + (1 - \sigma^i_k) A_i \bar{x}^i_{k|k-1} + B_i u^i_k. \quad \text{(A.2)}$$

By subtracting (A.2) from (A.1), the dynamics of the state estimation error ($e^i_{k|k-1} = \hat{x}^i_{k|k} - \bar{x}^i_{k|k-1}$) is as given in (18) for $\hat{e}^i_{k|k-1} = x^i_k - \hat{x}^i_{k|k-1}$ and $\hat{e}^i_{k|k} = x^i_k - \hat{x}^i_{k|k}$ where

$$\begin{aligned} \hat{e}^i_{k+1|k} &= A_i(I - L_i C_i)\hat{e}^i_{k|k-1} - A_i L_i v^i_k + w^i_k, \\ \hat{e}^i_{k+1|k+1} &= A_i \hat{e}^i_{k|k} + w^i_k - L_i(C_i \hat{e}^i_{k+1|k} + v^i_{k+1}). \end{aligned} \quad \text{(A.3)}$$

We know that when $\sigma^i_k = 1$, then the updated remote state estimation error is $\bar{e}^i_{k|k} = x^i_k - \bar{x}^i_{k|k} = \hat{e}^i_{k|k}$. However, when $\sigma^i_k = 0$, then $\bar{e}^i_{k|k} = x^i_k - \hat{x}^i_{k|k} + \hat{x}^i_{k|k} - \bar{x}^i_{k|k-1} = \hat{e}^i_{k|k} + e^i_{k|k-1}$. Using (18) and (A.3), $\bar{e}_{k|k}$ will have the following dynamics

$$\bar{e}^i_{k+1|k+1} = A_i \bar{e}^i_{k|k} + w^i_k \quad \text{(A.4)}$$

Therefore, we can express the updated remote state estimation error dynamics as follows

$$\bar{e}^i_{k+1|k+1} = \begin{cases} A_i \hat{e}^i_{k|k} + w^i_k, & \text{if } \sigma^i_k = 1 \\ A_i \bar{e}^i_{k|k} + w^i_k, & \text{otherwise.} \end{cases} \quad \text{(A.5)}$$

Denoting by $\Phi^i_{k|k} = \mathbb{E}[\bar{e}^i_{k|k} \bar{e}^{i\intercal}_{k|k} | \mathcal{J}^i_k]$ the covariance of the updated remote state estimation error at every time-step k, based on (A.5), we can conclude that it has the following dynamics

$$\Phi^i_{k+1|k+1} = (1 - \sigma^i_{k+1})(A_i^\intercal \Phi^i_{k|k} A_i + W^i) + \sigma^i_{k+1} \Theta_i.$$

From the fact that σ^i_{k+1} is independent from $\Phi^i_{k|k}$, we obtain

$$\mathbb{E}[\Phi^i_{k+1|k+1}] = (1 - q^i p^i)(A_i^\intercal \mathbb{E}[\Phi^i_{k|k}] A_i + W_i) + q^i p^i \Theta_i$$

Moreover, letting $\bar{\Phi}^i = \limsup_{k\to\infty} \mathbb{E}[\Phi^i_{k|k}]$, we find

$$\bar{\Phi}^i = (1 - q^i p^i)(A_i^\intercal \bar{\Phi}^i A_i + W_i) + q^i p^i \Theta_i.$$

This equation has the following closed form solution

$$\bar{\Phi}^i = \sum_{j=0}^{\infty} (1 - q^i p^i)^{j+1} (A_i^{j\intercal} W_i A_i^j) + q^i p^i (1 - q^i p^i)^j (A_i^{j\intercal} \Theta_i A_i^j)$$

which is bounded if $\varrho(\sqrt{1 - q^i p^i} A_i) < 1$ and holds by assumption of the lemma. On the other hand, the average quadratic performance (2) is given by [33, Ch. 5] as follows

$$J^i = \text{tr}(P_i W_i) + \limsup_{T\to\infty} \frac{1}{T} \sum_{k=0}^{T-1} \text{tr}(Y_i \mathbb{E}[\Phi^i_{k|k}]) \quad \text{(A.6)}$$

which can be expressed as

$$J^i = \text{tr}(P_i W_i) + \text{tr}(Y_i \limsup_{k\to\infty} \mathbb{E}[\Phi^i_{k|k}]) = \text{tr}(P_i W_i) + \text{tr}(Y_i \bar{\Phi}^i).$$

By substituting the solution of $\bar{\Phi}^i$ in the above equation, we arrive at the closed form of the average control performance for the PST policy in (15).

C Lemma 3

We consider both situations separately and drop the index i for simplicity.

1) $\delta^{st}_k = 0$: based on the Bayes law of conditional probability we have

$$f(e_{k|k}|\delta^{st}_k = 0, \mathcal{I}_k) = \frac{\Pr(\delta^{st}_k = 0|e_{k|k-1}, \mathcal{I}_k)}{\Pr(\delta^{st}_k = 0|\mathcal{I}_k)} f(e_{k|k-1}|\mathcal{I}_k). \quad \text{(A.7)}$$

Denote $z = e_{k|k-1}$, then based on the triggering policy (17) and considering $r_0 = \frac{1}{2} z^\intercal \Psi^{-1}_{k|k-1} z$ we have

$$\Pr(\delta^{st}_k = 0|z, \mathcal{I}_k) = \int_{r_0}^{\infty} \lambda_k e^{-\lambda_k r} dr = e^{-\frac{1}{2} z^\intercal \lambda_k \Psi^{-1}_{k|k-1} z}, \quad \text{(A.8)}$$

and

$$\begin{aligned} \Pr(\delta^{st}_k = 0|\mathcal{I}_k) &= \int_{z\in\mathbb{R}^n} \int_{r_0,}^{\infty} \frac{(\lambda_k e^{-\lambda_k r}) e^{-\frac{1}{2} z^\intercal \Psi^{-1}_{k|k-1} z}}{\det(2\pi\Psi_{k|k-1})^{\frac{1}{2}}} dr dz \\ &= \int_{z\in\mathbb{R}^n} \frac{e^{-\frac{1}{2} z^\intercal (1+\lambda_k) \Psi^{-1}_{k|k-1} z}}{\det(2\pi\Psi_{k|k-1})^{\frac{1}{2}}} dz = (1 + \lambda_k)^{-\frac{n}{2}}. \end{aligned} \quad \text{(A.9)}$$

Finally, by substituting (19), (A.8) and (A.9) into (A.7) we get

$$\begin{aligned} f(e_{k|k}|\delta^{st}_k = 0, \mathcal{I}_k) &= \frac{e^{-\frac{1}{2} z^\intercal (1+\lambda_k) \Psi^{-1}_{k|k-1} z}}{(1 + \lambda_k)^{-\frac{n}{2}} \det(2\pi\Psi_{k|k-1})^{\frac{1}{2}}} \\ &= \mathcal{N}(0, \frac{\Psi_{k|k-1}}{1 + \lambda_k}). \end{aligned}$$

Therefore, when $\delta_k^{st}=0$ the pdf of the updated state estimation error $e_{k|k}$ will remain Gaussian. Moreover, since $\nu_{k+1}=L(C\hat{e}_{k+1|k}+v_{k+1})$ in the dynamics of predicted state estimation error (18) is Gaussian, then the predicted state estimation error at the next time-step is also Gaussian as given in (21) where $\Phi=A^\intercal\Theta A-\Theta+W=\mathbb{E}[\nu_{k+1}\nu_{k+1}^\intercal]$ for all k.

2) $\delta_k^{st}=1,\ \rho_k=0$: in this case, the controller does not receive x_k, therefore

$$f(e_{k|k}|\delta_k^{st}=1,\rho_k=0,\mathcal{I}_k)= \frac{\Pr(\delta_k^{st}=1|e_{k|k-1},\rho_k=0,\mathcal{I}_k)}{\Pr(\delta_k^{st}=1|\rho_k=0,\mathcal{I}_k)}f(e_{k|k-1}|\mathcal{I}_k). \tag{A.10}$$

By using (A.8) and (A.9) we get

$$\Pr(\delta_k^{st}=1|z,\rho_k=0,\mathcal{I}_k)=1-e^{-\frac{1}{2}z^\intercal\lambda_k\Psi_{k|k-1}^{-1}z}, \tag{A.11}$$

and $p_k=\Pr(\delta_k^{st}=1|\rho_k=0,\mathcal{I}_k)=1-(1+\lambda_k)^{-\frac{n}{2}}$. Then by substitution into (A.10) we get the following

$$f(e_{k|k}|\delta_k^{st}=1,\rho_k=0,\mathcal{I}_k)=\frac{1}{p_k}\Big(\frac{e^{-\frac{1}{2}z^\intercal\Psi_{k|k-1}^{-1}z}}{\det(2\pi\Psi_{k|k-1})^{\frac{1}{2}}} -\frac{e^{-\frac{1}{2}z^\intercal(1+\lambda_k)\Psi_{k|k-1}^{-1}z}}{\det(2\pi\Psi_{k|k-1})^{\frac{1}{2}}}\Big).$$

Therefore, in case of a collision the pdf of the updated state estimation error will become the sum of two Gaussians

$$f(e_{k|k}|\delta_k^{st}=1,\rho_k=0,\mathcal{I}_k)=\frac{1}{p_k}\mathcal{N}(0,\Psi_{k|k-1}) -\frac{1-p_k}{p_k}\mathcal{N}(0,\frac{\Psi_{k|k-1}}{1+\lambda_k}).$$

Again with the same conclusion as the one presented for the case when $\delta_k^{st}=0$, the predicted state estimation error at the next time-step will be the sum of two Gaussians where their covariances follow (23).

D Theorem 2

First of all, we have to show that the CETT or equivalently STETT policy is in the class of admissible schedulers. The Kalman filter by the scheduler side and the remote state estimator follow (A.1) and (A.2), respectively. By subtracting (A.2) from (A.1), the dynamics of the state estimation error used by the scheduling policy (17) is determined by (18). Then, the scheduling law (17) can be represented as $\delta_k^i=g(\mathcal{R}_k^i)$ where $g(.)$ is an appropriate fixed function and $\mathcal{R}_k^i=\{v_t^i,w_t^i,r_t^i,\rho_t^i|t\in Z_0^{k-1}\}\cup\{v_k^i\}\cup\{x_0^i\}$ is a set of independent primitive random variables. Therefore, the CETT policy is in the class of admissible schedulers and the certainty equivalent control is optimal based on Theorem 1.

Now we have to find the estimated state in the controller for which we follow an induction arrangement. For the sake of simplicity, we consider a single control loop and drop the index i. Without loss of generality, let us assume that at $k=0$, $\sigma_0=1$ and find the state estimation at $k=1$ assuming $\sigma_1=0$. Then, $f(x_0|\sigma_0=1,\hat{x}_{0|0})=\mathcal{N}(\hat{x}_{0|0},\Theta)$ which can be concluded based on the properties of the Kalman filter. Since the remote state estimator is aware of the control inputs at all time-steps, the pdf of the predicted state at $k=1$ is

$$f(x_1|\sigma_0=1,\hat{x}_{0|0})=\mathcal{N}(\bar{x}_{1|0},\Gamma_{1|0}) \tag{A.12}$$

where $\bar{x}_{1|0}=A\hat{x}_{0|0}+Bu_0$ and $\Gamma_{1|0}=A^\intercal\Theta A+W$. Then the updated pdf of the remote state estimation at $k=1$ if $\sigma_1=0$ is determined by using Bayes law of conditional probability as follows

$$f(x_1|\sigma_0=1,\sigma_1=0,\hat{x}_{0|0}) =\frac{\Pr(\sigma_1=0|\sigma_0=1,\hat{x}_{0|0},x_1)f(x_1|\sigma_0=1,\hat{x}_{0|0})}{\Pr(\sigma_1=0|\sigma_0=1,\hat{x}_{0|0})}. \tag{A.13}$$

Moreover, we have

$$\Pr(\sigma_1=0|\sigma_0=1,\hat{x}_{0|0},x_1)=\Pr(\delta_1=0|\sigma_0=1,\hat{x}_{0|0},x_1) +\Pr(\rho_1=0)\Pr(\delta_1=1|\sigma_0=1,\hat{x}_{0|0},x_1). \tag{A.14}$$

Let $\hat{z}_1=x_1-\hat{x}_{1|1}$, $\bar{z}_1=x_1-\bar{x}_{1|0}$ and $z_1=\hat{x}_{1|1}-\bar{x}_{1|0}$ where $f(\hat{z}_1)=\mathcal{N}(0,\Theta)$ and $\mathbb{E}[z_1z_1^\intercal|\sigma_0=1]=\Psi_{1|0}$ for $\Psi_{1|0}=\Phi=A^\intercal\Theta A-\Theta+W$. Then

$$\Pr(\delta_1=0|\sigma_0=1,\hat{x}_{0|0},x_1)=\Pr(\delta_1=0|\sigma_0=1,\bar{x}_{1|0},x_1)=$$

$$\Pr(\delta_1=0|\sigma_0=1,\bar{z}_1)=\int_{\hat{z}_1\in\mathbb{R}^n}\int_{r_0,}^{\infty}\frac{\lambda_1e^{-\lambda_1r}e^{-\frac{1}{2}\hat{z}_1^\intercal\Theta^{-1}\hat{z}_1}}{\det(2\pi\Theta)^{\frac{1}{2}}}drd\hat{z}_1$$

$$=\int_{\hat{z}_1\in\mathbb{R}^n}\frac{e^{-\frac{1}{2}(\bar{z}_1-\hat{z}_1)^\intercal\lambda\Psi_{1|0}^{-1}(\bar{z}_1-\hat{z}_1)-\frac{1}{2}\hat{z}_1^\intercal\Theta^{-1}\hat{z}_1}}{\det(2\pi\Theta)^{\frac{1}{2}}}d\hat{z}_1$$

for $r_0=\frac{1}{2}z_1^\intercal\Psi_{1|0}^{-1}z_1=\frac{1}{2}(\bar{z}_1-\hat{z}_1)^\intercal\Psi_{1|0}^{-1}(\bar{z}_1-\hat{z}_1)$. We can simplify the above equation by using the following equality

$$\begin{aligned}&(\bar{z}_1-\hat{z}_1)^\intercal\lambda_1\Psi_{1|0}^{-1}(\bar{z}_1-\hat{z}_1)+\hat{z}_1^\intercal\Theta^{-1}\hat{z}_1\\&=(\hat{z}_1-\bar{z}_1')^\intercal(\lambda_1\Psi_{1|0}^{-1}+\Theta^{-1})(\hat{z}_1-\bar{z}_1')+\bar{z}_1^\intercal\Pi_{1|0}^{-1}\bar{z}_1\end{aligned}$$

where $\bar{z}_1'=(\lambda_1\Psi_{1|0}^{-1}+\Theta^{-1})^{-1}\lambda_1\Psi_{1|0}^{-1}\bar{z}_1$ and

$$\begin{aligned}\Pi_{1|0}&=\big(\lambda_1\Psi_{1|0}^{-1}-\lambda_1\Psi_{1|0}^{-1}(\lambda_1\Psi_{1|0}^{-1}+\Theta^{-1})^{-1}\lambda_1\Psi_{1|0}^{-1}\big)^{-1}\\&=\frac{1}{\lambda_1}\Psi_{1|0}+\Theta.\end{aligned}$$

Then

$$\Pr(\delta_1=0|\sigma_0=1,\hat{x}_{0|0},x_1)=\xi_1e^{-\frac{1}{2}\bar{z}_1^\intercal\Pi_{1|0}^{-1}\bar{z}_1} \tag{A.15}$$

where $\xi_1=1/\big(\det(\lambda_1\Psi_{1|0}^{-1}+\Theta^{-1})\det(\Theta)\big)$. Moreover,

$$\Pr(\delta_1=1|\sigma_0=1,\hat{x}_{0|0},x_1)=1-\xi_1e^{-\frac{1}{2}\bar{z}_1^\intercal\Pi_{1|0}^{-1}\bar{z}_1}. \tag{A.16}$$

First substitute (A.15) and (A.16) into (A.14) which results in

$$\Pr(\sigma_1=0|\sigma_0=1,\hat{x}_{0|0},x_1)=1-q_1+q_1\xi_1e^{-\frac{1}{2}\bar{z}_1^\intercal\Pi_{1|0}^{-1}\bar{z}_1}$$

where $q_1=\Pr(\rho_1=1)$, then substitute the result into (A.13) which results in the following

$$\begin{aligned}&f(x_1|\sigma_0=1,\sigma_1=0,\hat{x}_{0|0})\\&\quad=\frac{1-q_1+q_1\xi_1e^{-\frac{1}{2}\bar{z}_1^\intercal\Pi_{1|0}^{-1}\bar{z}_1}}{1-q_1p_1}\frac{e^{-\frac{1}{2}\bar{z}_1^\intercal\Gamma_{1|0}^{-1}\bar{z}_1}}{\det(2\pi\Gamma_{1|0})^{\frac{1}{2}}}\\&\quad=\sum_{i=1}^{2}\beta_i'\mathcal{N}(\bar{x}_{1|0},\Gamma_{1|1}^i)\end{aligned} \tag{A.17}$$

where $\sum_{i=1}^{2}\beta_i'=1$. As it can be seen, at the first time-step after the successful transmission the updated state estimation pdf is the sum of two Gaussian terms with different covariances. The total covariance of the estimation is affected by q_1 which is the probability that the network is available. However, the mean values of the Gaussian terms are equal to the one obtained at the prediction stage (A.12) and do not depend on q_1, i.e. $\bar{x}_{1|1}=\mathbb{E}[x_1|\sigma_0=1,\sigma_1=0,\hat{x}_{0|0}]=\bar{x}_{1|0}$. Now assume $\sigma_k=0,\ \forall k\in Z_1^{t+1}$, then the updated state estimation at $k=t+1$ is determined as follows

$$f(x_{t+1}|\nu_t,\sigma_{t+1}=0,\hat{x}_{0|0}) = \frac{\Pr(\sigma_{t+1}=0|\nu_t,\hat{x}_{0|0},x_{t+1})f(x_{t+1}|\nu_t,\hat{x}_{0|0})}{\Pr(\sigma_{t+1}=0|\nu_t,\hat{x}_{0|0})} \tag{A.18}$$

where $\nu_t=\{\sigma_0=1,\sigma_1=\cdots=\sigma_t=0\}$. Let us define $c=\inf\{k|\delta_k=1\wedge k\in Z_1^t\}$ as the first time-step after the last successful transmission where data collision happens, otherwise, $c=0$. Then we can partition the set ν_t into several mutually exclusive sets as ν_t^c, where for every $c\in Z_1^t$

$$\nu_t^c=\big\{\sigma_0=1,\delta_1=\cdots=\delta_{c-1}=0,\{\delta_c=1,\rho_c=0\},\ \sigma_{c+1}=\cdots=\sigma_t=0\big\}$$

and $\nu_t^0=\{\sigma_0=1,\delta_1=\cdots=\delta_t=0\}$. Therefore,

$$\begin{aligned}&\Pr(\sigma_{t+1}=0|\nu_t,\hat{x}_{0|0},x_{t+1})f(x_{t+1}|\nu_t,\hat{x}_{0|0})\\&=\sum_{j=0}^{t}\Pr(\sigma_{t+1}=0|\nu_t^j,\hat{x}_{0|0},x_{t+1})\Pr(c=j)f(x_{t+1}|\nu_t^j,\hat{x}_{0|0}).\end{aligned} \tag{A.19}$$

According to the operation mechanism of the CETT policy, the predicted state estimation has the following distribution

$$f(x_{t+1}|\nu_t^j,\hat{x}_{0|0})=\begin{cases}\mathcal{N}(\bar{x}_{t+1|t},\Gamma^0_{t+1|t}), & \text{if } j=0\\ \sum_{i=1}^{2}\mathcal{N}(\bar{x}_{t+1|t},\Gamma^{j,i}_{t+1|t}), & \text{otherwise}\end{cases}$$

where, according to the induction assumption, the mean values of all Gaussian terms are equal and determined as $\bar{x}_{t+1|t}=A\bar{x}_{t|t-1}+Bu_t$ and $\Gamma^{j,i}_{t+1|t}=A\Gamma^{j,i}_{t|t}A^\intercal+W$ is their covariance.

Moreover, based on the triggering policy, after the first collision instance which results in two Gaussian terms in the pdf of the state estimation, the scheduling policy switches to the purely stochastic policy where $\Pr(\delta_{t+1}=0|\nu_t^j,\hat{x}_{0|0},x_{t+1})=1-p_{t+1},\ \forall j\neq 0$. However, following the same procedure as the one for $t=1$,

$$\Pr(\sigma_{t+1}=0|\nu_t^0,\hat{x}_{0|0},x_{t+1})=1-q_{t+1}+q_{t+1}\xi_{t+1}e^{-\frac{1}{2}\bar{z}_{t+1}^\intercal\Pi^{-1}_{t+1|t}\bar{z}_{t+1}}.$$

where $\xi_{t+1}=1/\big(\det(\lambda_{t+1}\Psi^{-1}_{t+1|t}+\Theta^{-1})\det(\Theta)\big)$, $\bar{z}_{t+1}=x_{t+1}-\bar{x}_{t+1|t}$ and $\Pi_{t+1|t}=\frac{1}{\lambda_{t+1}}\Psi_{t+1|t}+\Theta$. Then by substituting the last two expressions into (A.19) and then into (A.18), we can arrive at the following

$$f(x_{t+1}|\nu_t,\sigma_{t+1}=0,\hat{x}_{0|0}) = \frac{\sum_{j=0}^{t}\sum_{i=1}^{2}g(i,j,p_{t+1},q_{t+1})\mathcal{N}(\bar{x}_{t+1|t},\Gamma^{j,i}_{t+1|t+1})}{1-q_{t+1}p_{t+1}}$$

where $g(i,j,p_{t+1},q_{t+1})$ is a scalar function. Therefore, at $k=t+1$ the number of Gaussian terms is equal to $2(t+1)$. However, the mean of all these terms are equal and not affected by the kind of scheduling policy (PST or STETT), the triggering probability p_{t+1}, or the collision probability q_{t+1}, that is in line with the induction assumption. Therefore, $\bar{x}_{t+1|t+1}=\bar{x}_{t+1|t}$ when $\sigma_{t+1}=1$ and (13) still holds when the scheduler is operating based on CETT policy and the result follows.

E Theorem 3

Let us consider a single control loop and drop the index i for simplicity. Consider $N\geq 1$ as the sampling period for every two successive successful transmission, then the average control performance (2) can be written as

$$\begin{aligned}J_a&=\operatorname{tr}(PW)+\frac{1}{\mathbb{E}[N]}\mathbb{E}[\sum_{t=0}^{N-1}\operatorname{tr}(Y\Gamma^a_{t|t})]\\&=\operatorname{tr}(PW)+\sum_{v=1}^{\infty}\frac{\Pr(N=v)}{\mathbb{E}[N]}\sum_{t=0}^{v-1}\operatorname{tr}(Y\Gamma^a_{t|t})\end{aligned} \tag{A.20}$$

which can be established using Wald's identity as in [11] where $\Gamma^a_{t|t}=\mathbb{E}[\bar{e}_{t|t}\bar{e}^\intercal_{t|t}|\hat{x}_{0|0},I_t]$ for $I_t=\{\sigma_k=0|k\in Z_1^t\}$ and $a\in\{ps,\mu\}$. Given the assumption that both scheduling policies are triggering with the same probabilities at every time-step, to prove Theorem 3, it is just needed to prove

$$\hat{J}^v_\mu<\hat{J}^v_{ps} \tag{A.21}$$

where $\hat{J}^v_a=\sum_{t=1}^{v-1}\operatorname{tr}(Y\Gamma^a_{t|t})$ for $a\in\{ps,\mu\}$ and $v\in\mathbb{N}_{\geq 2}$ (at $t=0$ both policies result in the same cost values). For the following analysis, let us define

$$\big(m(i),l(i)\big)=\begin{cases}(0,0), & \text{if } i=1\\(1,0), & \text{if } i=2\\(0,1), & \text{if } i=3\end{cases} \tag{A.22}$$

where $m(i)$ and $l(i)$ are possible values for the triggering variable and the availability of the communication network for each user, respectively, which result in no successful data transmission to the controller.

We also need the following technical proposition.

Proposition 1: Consider

$$\Psi^a_{1|1}(i)=\mathbb{E}[e_{1|1}e^\intercal_{1|1}|\hat{x}_{0|0},\delta^a_1=m(i),\rho^a_1=l(i)]$$

as the updated covariance of the state estimation error in the scheduler at $t=1$ for every transmission epoch with $v>2$ where $i\in\{1,2,3\}$ and $m(i)$, $l(i)$ follow (A.22). Then

$$\sum_{t=2}^{v-1}\operatorname{tr}(Y\Gamma^a_{t|t})|[\hat{x}_{0|0},\delta^a_1=m(i),\rho_1=l(i)]=\operatorname{tr}\big(L_a(\Psi^a_{1|1}(i)+\Theta)\big)+\hat{J}^v_a$$

where L_a for $a\in\{\mu,ps\}$ is a positive definite matrix such that $L_\mu<L_{ps}$. □

The proof of Proposition 1 is provided after the proof of Theorem 3.

E1 Proof of Theorem 3: We need to establish (A.21), which we shall proof by using induction on v. Suppose $v=2$, then for every $a\in\{\mu,ps\}$ we have

$$\hat{J}_a^2=\text{tr}(Y\Gamma_{1|1}^a)=\sum_{i=1}^{3}\text{tr}\big(Y\bar{\Gamma}_{1|1}^a(i)\big)S^a(i)$$

where

$$\begin{aligned}\bar{\Gamma}_{1|1}^a(i)=&\mathbb{E}[\bar{e}_{1|1}\bar{e}_{1|1}^\intercal|\hat{x}_{0|0},\delta_1^a=m(i),\rho_1=l(i)],\\ S^a(i)=&\text{Pr}(\delta_1^a=m(i),\rho_1=l(i))\end{aligned}$$

for every index value of $i\in\{1,2,3\}$ for which $m(i)$ and $l(i)$ are defined in (A.22). In Tables I and II we determine the values of these terms. Consider $r_0:=\text{tr}(Y\Theta)$, $r_1:=\text{tr}(Y\Phi)$ for $\Phi=A\Theta A^\intercal-\Theta+W$, $r_2:=\frac{r_1}{1+\lambda_1}$ and $r_3:=\frac{1}{p_1}r_1-\frac{1-p_1}{p_1}r_2$ which are used in the tables.

Table I: Performance terms of the CETC policy when $v=2$

i	δ_1^μ	ρ_1	S^μ	$\text{tr}(Y\bar{\Gamma}_{1\|1}^\mu)$
1	0	0	$(1-q_1)(1-p_1)$	r_0+r_2
2	1	0	$(1-q_1)p_1$	r_0+r_3
3	0	1	$q_1(1-p_1)$	r_0+r_2

Table II: Performance terms of the PST policy when $v=2$

i	δ_1^{ps}	ρ_1	S^{ps}	$\text{tr}(Y\bar{\Gamma}_{1\|1}^{ps})$
1,2	0 or 1	0	$(1-q_1)$	r_0+r_1
3	0	1	$q_1(1-p_1)$	r_0+r_1

As an example, for the CETC policy when $i=2$, we have $\delta_1^\mu=1$ and $\rho_1=0$. This condition results in a collision in which the updated state error covariance is determined as

$$\bar{\Gamma}_{1|1}^\mu|[i=2]=\Theta+\frac{1}{p_1}(1-\frac{1-p_1}{1+\lambda_1})\Phi$$

and by substitution into $\text{tr}(Y\bar{\Gamma}_{1|1}^\mu)$, it results in r_0+r_3 as written in Table I. However, for the PST policy, in case the network is not available, the updated state error covariance is $\Gamma_{1|1}^{ps}|[i=1,2]=\Theta+\Phi$ which results in r_0+r_1 as the control performance. Similar reasonings can be applied to the other situations.

Note that

$$\begin{aligned}\hat{J}_\mu^2&=(1-q_1)(r_0+r_1)+q_1(1-p_1)(r_0+r_2),\\ \hat{J}_{ps}^2&=(1-q_1)(r_0+r_1)+q_1(1-p_1)(r_0+r_1)\end{aligned}$$

from which it is clear that $\hat{J}_\mu^2<\hat{J}_{ps}^2$ since $r_2<r_1$.
Now assume (A.21) holds for $v=z$, i.e. $\hat{J}_\mu^z<\hat{J}_{ps}^z$, then we should prove the same inequality for $v=z+1$. We have

$$\hat{J}_a^{z+1}=\sum_{i=1}^{3}\Big(\text{tr}\big(Y\bar{\Gamma}_{1|1}^a(i)\big)+\sum_{t=2}^{z}\text{tr}(Y\Gamma_{t|t}^a)|[i]\Big)S^a(i)$$

and by using Proposition 1, we can simplify this equation as follows

$$\hat{J}_a^{z+1}=\sum_{i=1}^{3}\big(\text{tr}(Y\bar{\Gamma}_{1|1}^a(i))+\text{tr}\big(L_a(\Psi_{1|1}^a(i)+\Theta)\big)+\hat{J}_a^v\big)S^a(i),$$

Therefore, by considering the assumption of induction, in order to prove $\hat{J}_\mu^{z+1}<\hat{J}_{ps}^{z+1}$, we just need to prove $\hat{j}_\mu^{z+1}<\hat{j}_{ps}^{z+1}$ where for $a\in\{\mu,ps\}$, $\hat{j}_a^{z+1}=\sum_{i=1}^{3}C^a(i)S^a(i)$, in which $C^a(i)=\text{tr}\big(Y\bar{\Gamma}_{1|1}^a(i)\big)+\text{tr}\big(L_a\Psi_{1|1}^a(i)\big)$. Then let us denote $s_1:=\text{tr}(L_{ps}\Phi)$, $s_2:=\text{tr}(L_\mu\Phi)/(1+\lambda_1)$, $s_3:=\frac{1}{p_1}(1-\frac{1-p_1}{1+\lambda_1})\text{tr}(L_\mu\Phi)$ and $s_4:=\text{tr}(L_\mu\Phi)$. Based on Proposition 1

$$s_2<s_4<s_1. \tag{A.23}$$

In Tables III and IV the values of $C^a(i)$ are given for the CETC and the PST policies, respectively.

Table III: Performance terms of the CETC policy

i	δ_1^μ	ρ_1	$S^\mu(i)$	$C^\mu(i)$
1	0	0	$(1-q_1)(1-p_1)$	$r_0+r_2+s_2$
2	1	0	$(1-q_1)p_1$	$r_0+r_3+s_3$
3	0	1	$q_1(1-p_1)$	$r_0+r_2+s_2$

Table IV: Performance terms of the PST policy

i	δ_1^{ps}	ρ_1	$S^{ps}(i)$	$C^{ps}(i)$
1	0 or 1	0	$(1-q_1)$	$r_0+r_1+s_1$
2	0	1	$q_1(1-p_1)$	$r_0+r_1+s_1$

Then we have

$$\begin{aligned}\hat{j}_\mu^{z+1}&=(1-q_1)(r_0+r_1+s_4)+q_1(1-p_1)(r_0+r_2+s_2),\\ \hat{j}_{ps}^{z+1}&=(1-q_1)(r_0+r_1+s_1)+q_1(1-p_1)(r_0+r_1+s_1)\end{aligned}$$

and by using the inequalities given in (A.23) and $r_2<r_1$ we can infer $\hat{j}_\mu^{z+1}<\hat{j}_{ps}^{z+1}$ which concludes the proof.

E2 Proof of Proposition 1: This proposition actually considers the propagation of the first time-step's state estimation error covariance in the future time-steps during every transmission epoch. We know that when $\sigma_t=0$ for $t\in Z_1^{v-1}$ during every transmission epoch, $\bar{e}_{t|t}=\hat{e}_{t|t}+e_{t|t}$ where $\hat{e}_{t|t}\sim\mathcal{N}(0,\Theta)$.

For the PST policy, we know that $\bar{e}_{1|1}\sim\mathcal{N}(0,\Psi_{1|1}^{ps}+\Theta)$ which will increase the covariance of the future errors as $A^{\intercal t-1}(\Psi_{1|1}^{ps}+\Theta)A^{t-1}$ for all $t\in Z_2^{v-1}$. Therefore, the total amount of increase of the cost function during every transmission epoch due to the first time-step state estimation error will be $\delta\hat{J}_{ps}=\sum_{j=2}^{v-1}\text{tr}\big(A^{\intercal j-1}YA^{j-1}(\Psi_{1|1}^{ps}+\Theta)\big)$ which results in $L_{ps}=\sum_{j=1}^{v-2}A^{\intercal j}YA^j$. Now let us consider the CETC policy and denote

$$\beta(l)=\begin{cases}(1-p)^{\frac{2}{n}}, & \text{if } l<0\\ \frac{1-(1-p)^{1+\frac{2}{n}}}{p}, & \text{if } l=0\\ 1, & \text{otherwise.}\end{cases}$$

Suppose that at $t=1$ the first collision has occurred. Then from the next time-step, the scheduler follows the PST policy where the increase in the value of the covariance will be as the one obtained for the PST policy, i.e. $\delta\hat{J}_\mu^{c=1}=\sum_{j=2}^{v-1}\text{tr}\big(A^{\intercal j-1}YA^{j-1}(\Psi_{1|1}^{\mu}+\Theta)\big)$ which results in $L_\mu^{c=1}=\sum_{j=1}^{v-2}A^{\intercal j}YA^j$. Now suppose that collision occurs at $t=2$, then we can show that $L_\mu^{c=2}=\sum_{j=1}^{v-2}\beta(0)A^{\intercal j}YA^j$ and if collision occurs at $t=k>2$, then

$$L_\mu^{c=k}=\sum_{j=1}^{v-2}\prod_{l=2-k}^{2-k+j-1}\beta(l)A^{\intercal j}YA^j.$$

Therefore,

$$L_\mu = \sum_{k=1}^{v-1} p(1-p)^{k-1} L_{c=k}^{\mu} = \sum_{j=1}^{v-2} \alpha(j) A^{\intercal j} Y A^j$$

where $\alpha(j) = p + \sum_{k=2}^{v-1} p(1-p)^{k-1} \prod_{l=2-k}^{2-k+j-1} \beta(l)$. In order to prove $L_\mu \leq L_{ps}$, it is just needed to prove $\alpha(j) \leq 1$ for all $j \in Z_1^{v-2}$. For an arbitrary j we have

$$\begin{aligned}\alpha(j) &= p + \beta(0)p(1-p) + \beta(-1)\beta(0)p(1-p)^2 + \dots \\ &\quad + \beta(-1)^{j-1}\beta(0)p(1-p)^j + \beta(-1)^j p(1-p)^{j+1} \\ &\quad + \dots + \beta(-1)^j p(1-p)^{v-2} = \dots \\ &= 1 - (1-p)^{v+\frac{2j}{n}-2} < 1\end{aligned}$$

which concludes our statement and proves the proposition.

F Lemma 4

We know $\eta^i = p^i \prod_{j=1, j\neq i}^{m} (1-p^j)$, then

$$\begin{aligned}\sum_{j=1}^{m} U^j(\eta^j) &= \sum_{j=1}^{m} \big(\log(p^j)^{c^j} + \sum_{i=1, i\neq j}^{m} \log(1-p^i)^{c^j} \big) \\ &= \sum_{j=1}^{m} \log \big((p^j)^{c^j} (1-p^j)^{\sum_{i=1, i\neq j}^{m} c^i} \big)\end{aligned}$$

which indicates that

$$p^{j*} = \arg \max_{p^j \in [0,1]} (p^j)^{c^j} (1-p^j)^{\sum_{i=1, i\neq j}^{m} c^i}$$

which results in (28).

References

[1] W. Heemels, K. H. Johansson, and P. Tabuada, "An introduction to event-triggered and self-triggered control," in *Decision and Control (CDC), 2012 IEEE 51st Annual Conference on*. IEEE, 2012, pp. 3270–3285.

[2] W. H. Heemels, M. Donkers, and A. R. Teel, "Periodic event-triggered control for linear systems," *IEEE Transactions on Automatic Control*, vol. 58, no. 4, pp. 847–861, 2013.

[3] T. Soleymani, S. Hirche, and J. S. Baras, "Event-triggered output-feedback h_∞ control with minimum directed information," in *Decision and Control (CDC), 2017 IEEE 56th Annual Conference on*. IEEE, 2017, pp. 6088–6094.

[4] C. Nowzari, E. Garcia, and J. Cortes, "Event-triggered communication and control of network systems for multi-agent consensus," *arXiv preprint arXiv:1712.00429*, 2017.

[5] B. A. Khashooei, D. Antunes, and W. Heemels, "A consistent threshold-based policy for event-triggered control," *IEEE Control Systems Letters*, 2018.

[6] X. Wang and M. D. Lemmon, "Event-triggering in distributed networked control systems," *IEEE Transactions on Automatic Control*, vol. 56, no. 3, p. 586, 2011.

[7] A. Molin and S. Hirche, "A bi-level approach for the design of event-triggered control systems over a shared network," *Discrete Event Dynamic Systems*, vol. 24, no. 2, pp. 153–171, 2014.

[8] M. H. Mamduhi, D. Tolić, A. Molin, and S. Hirche, "Event-triggered scheduling for stochastic multi-loop networked control systems with packet dropouts," in *Decision and Control (CDC), 2014 IEEE 53rd Annual Conference on*. IEEE, 2014, pp. 2776–2782.

[9] B. Demirel, V. Gupta, D. E. Quevedo, and M. Johansson, "Threshold optimization of event-triggered multi-loop control systems," in *Discrete Event Systems (WODES), 2016 13th International Workshop on*. IEEE, 2016, pp. 203–210.

[10] J. F. Kurose, *Computer networking: A top-down approach featuring the internet, 3/E*. Pearson Education India, 2005.

[11] F. D. Brunner, D. Antunes, and F. Allgöwer, "Stochastic thresholds in event-triggered control: A consistent policy for quadratic control," *Automatica*, vol. 89, pp. 376–381, 2018.

[12] D. Han, J. Wu, Y. Mo, and L. Xie, "On stochastic sensor network scheduling for multiple processes," *IEEE Transactions on Automatic Control*, 2017.

[13] A. Cervin and T. Henningsson, "Scheduling of event-triggered controllers on a shared network," in *Decision and Control, 2008. CDC 2008. 47th IEEE Conference on*. IEEE, 2008, pp. 3601–3606.

[14] R. Blind and F. Allgöwer, "On time-triggered and event-based control of integrator systems over a shared communication system," *Mathematics of Control, Signals, and Systems*, vol. 25, no. 4, pp. 517–557, 2013.

[15] M. Xia, V. Gupta, and P. J. Antsaklis, "Networked state estimation over a shared communication medium," *IEEE Transactions on Automatic Control*, vol. 62, no. 4, pp. 1729–1741, 2017.

[16] C. Ramesh, H. Sandberg, L. Bao, and K. H. Johansson, "On the dual effect in state-based scheduling of networked control systems," in *American Control Conference (ACC), 2011*, June 2011, pp. 2216–2221.

[17] M. H. Mamduhi, M. Kneissl, and S. Hirche, "Decentralized event-triggered medium access control for networked control systems," in *Decision and Control (CDC), 2016 IEEE 55th Conference on*. IEEE, 2016, pp. 513–519.

[18] M. H. Mamduhi, A. Molin, D. Tolić, and S. Hirche, "Error-dependent data scheduling in resource-aware multi-loop networked control systems," *Automatica*, vol. 81, pp. 209–216, 2017.

[19] A. Molin and S. Hirche, "On the optimality of certainty equivalence for event-triggered control systems," *IEEE Transactions on Automatic Control*, vol. 58, no. 2, pp. 470–474, 2013.

[20] A. Goldenshluger and L. Mirkin, "On minimum-variance event-triggered control," *IEEE control systems letters*, vol. 1, no. 1, pp. 32–37, 2017.

[21] V. Gupta, B. Hassibi, and R. M. Murray, "Optimal lqg control across packet-dropping links," *Systems & Control Letters*, vol. 56, no. 6, pp. 439–446, 2007.

[22] X. Meng and T. Chen, "Optimal sampling and performance comparison of periodic and event based impulse control," *IEEE Transactions on Automatic Control*, vol. 57, no. 12, pp. 3252–3259, 2012.

[23] D. Antunes and W. Heemels, "Rollout event-triggered control: Beyond periodic control performance," *IEEE Transactions on Automatic Control*, vol. 59, no. 12, pp. 3296–3311, 2014.

[24] S. Al-Areqi, D. Görges, and S. Liu, "Event-based networked control and scheduling codesign with guaranteed performance," *Automatica*, vol. 57, pp. 128–134, 2015.

[25] D. Shi, T. Chen, and L. Shi, "On set-valued kalman filtering and its application to event-based state estimation," *IEEE Transactions on Automatic Control*, vol. 60, no. 5, pp. 1275–1290, 2015.

[26] S. Linsenmayer and F. Allgöwer, "Performance oriented triggering mechanisms with guaranteed traffic characterization for linear discrete-time systems," in *2018 European Control Conference (ECC)*. IEEE, 2018, pp. 1474–1479.

[27] M. H. Balaghi, D. J. Antunes, M. H. Mamduhi, and S. Hirche, "A decentralized consistent policy for event-triggered control over a shared contention-based network," in *2018 IEEE Conference on Decision and Control (CDC)*. IEEE, 2018, pp. 1719–1724.

[28] M. H. Balaghi I, D. J. Antunes, M. H. Mamduhi, and S. Hirche, "An optimal lqg controller for stochastic event-triggered scheduling over a lossy communication network," *IFAC-PapersOnLine*, vol. 51, no. 23, pp. 58–63, 2018.

[29] J.-W. Lee, M. Chiang, and R. Calderbank, "Jointly optimal congestion and contention control based on network utility maximization," *IEEE Communications letters*, vol. 10, no. 3, pp. 216–218, 2006.

[30] A. Molin, "Optimal event-triggered control with communication constraints," Dissertation, Technical University of Munich, Munich, 2014.

[31] D. J. Antunes and B. A. Khashooei, "Consistent event-triggered methods for linear quadratic control," in *Decision and Control (CDC), 2016 IEEE 55th Conference on*. IEEE, 2016, pp. 1358–1363.

[32] Y. Bar-Shalom and E. Tse, "Dual effect, certainty equivalence, and separation in stochastic control," *IEEE Transactions on Automatic Control*, vol. 19, no. 5, pp. 494–500, 1974.

[33] D. P. Bertsekas, *Dynamic Programming and Optimal Control*, 3rd ed. Athena Scientific, 2005.

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